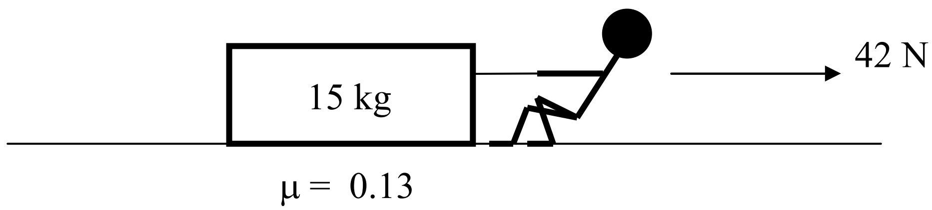
Applications of Friction Force

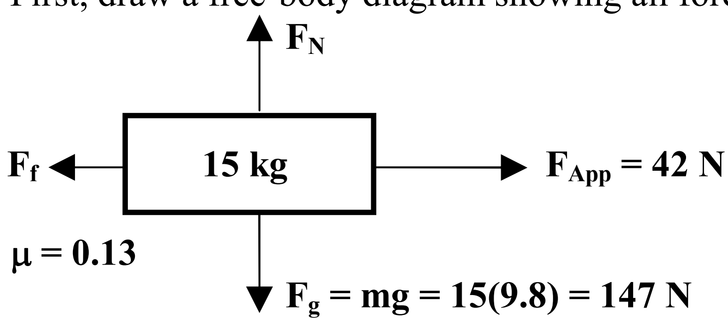
To start, think about objects that are moved along a surface, and the net force acts parallel to that surface. This is similar to problems you encountered in Physics 11.

Consider the following: a 15 kg mass is pulled along a horizontal surface with a force of 42 N. A coefficient of friction of $\mu = 0.13$ exists between the mass and the surface.



To find the acceleration of the mass, follow these procedures:

- First, draw a free-body diagram showing all forces acting on the mass:



- From inspection, $\mathbf{F_{Net}}$ acts horizontally to the right: $\mathbf{F_{Net} = 42 - F_f}$

→ to find $\mathbf{F_f}$, use $\mathbf{F_f = \mu F_N}$

where $\mathbf{F_N = F_g = 15(9.8) = 147\ N}$

→ $\mathbf{F_f = .13(147) = 19.1\ N}$

- Now solve for $\mathbf{F_{Net}}$ and **a:**

→$\mathbf{F_{Net} = 42 - 19.1 = 23\ N}$

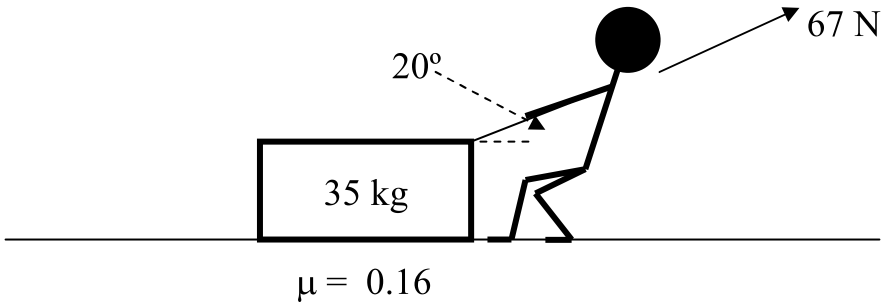
→ $\mathbf{F_{Net} = ma}$ →$\mathbf{23 = 15a}$ →$\mathbf{a = 1.5\ m/s^2}$

Example #6. Find the acceleration of a 4.0 kg mass when a 20 N force is applied and the coefficient of friction between mass and surface is 0.10.

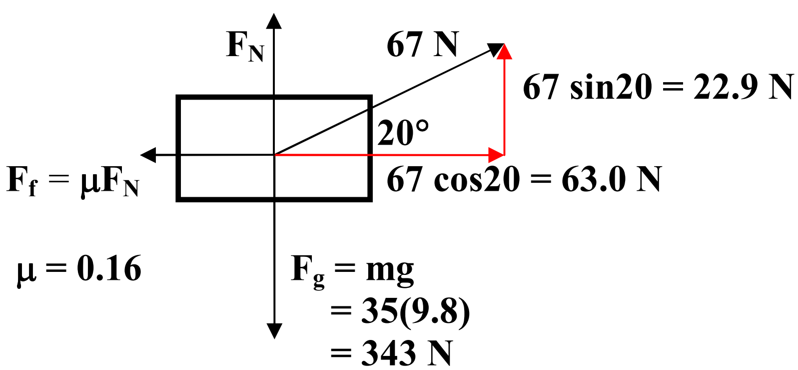
(see Dynamics Ex 6 for answer)

Next, let's consider an angled pull, involving friction. An angled applied force affects the normal force and therefore <u>also affects friction</u>. An angled force upwards *reduces* the magnitude of both forces, while an angled force downwards *increases* those same quantities.

Suppose you have a force of 67 N pulling at an angle of 20º up on a mass of 35 kg in order to move it along a horizontal floor. If the coefficient of friction is 0.16, find the acceleration.



- Start with a f.b. diagram, and break the applied force of **67 N** into components: (note that vectors are <u>not</u> drawn to scale)



- As usual, to find '**a**' we must first determine $\mathbf{F_{Net}}$, which is to the right, and horizontal.

- By examining the horizontal vectors in the diagram, $\mathbf{F_{Net} = 63.0 - F_f}$

- We must find $\mathbf{F_f}$, where $\mathbf{F_f = \mu F_N}$
  → so we need to first find $\mathbf{F_N}$ by analyzing the *vertical* force vectors, which are perpendicular to the surface:

$$\mathbf{F_N + 22.9 = F_g} \quad \rightarrow \quad \mathbf{F_N = 343 - 22.9 = 320\ N}$$

→ now calculate $\mathbf{F_f}$: $\mathbf{F_f = \mu F_N = 0.16(320) = 51.2\ N}$

- At this point, we can find both $\mathbf{F_{Net}}$ and **a**:

→F_{Net} = 63.0 – F_f = 63.0 – 51.2 = 11.8 N

→F_{Net} = ma →11.8 = 35a →a = 0.34 m/s^2

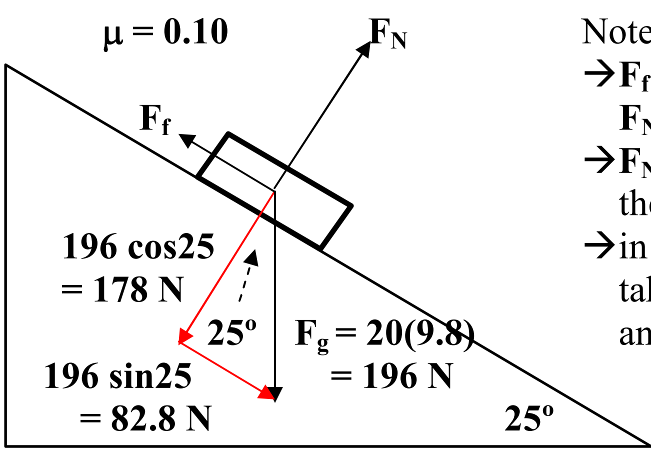
Keep in mind that for any problems involving Newton's Laws, you may have to deal with some or all of the following: force *components*, $\mathbf{F_N}$, $\mathbf{F_f}$, $\mathbf{F_{Net}}$, and/or **a.**

Example #7. A force of 75 N pushes <u>down</u> at an angle of 15º on a mass of 25 kg. If the coefficient of friction is 0.15, find the acceleration.

(see Dynamics Ex 7 for answer)

Slightly more difficult problems involve objects placed on a sloped surface. For this situation, consider a 20.0 kg mass, set on a smooth inclined ramp of μ = 0.10. What is its acceleration if the angle of the slope is 25º?

- First, review how pulling and pressing forces are found on an incline:



Note the following:
→$\mathbf{F_f}$ is parallel to the surface, while $\mathbf{F_N}$ is perpendicular
→$\mathbf{F_{Net}}$ will act downslope, parallel to the surface
→in order to find $\mathbf{F_N}$, $\mathbf{F_f}$ and $\mathbf{F_{Net}}$ we must take components of $\mathbf{F_g}$ that are parallel and perpendicular to the surface

Note the following items shown on the diagram:

a) Calculation of weight $\mathbf{F_g}$
b) The parallel and perpendicular vector components of $\mathbf{F_g}$, along with the angle within the components triangle
c) Calculation of the parallel component of $\mathbf{F_g}$, pulling the mass down-slope; this is **mg sinθ**
d) Calculation of the perpendicular component of $\mathbf{F_g}$, pressing the mass into the ramp; this is **mg cosθ**

- Second, examine *perpendicular* vectors to find $\mathbf{F_N}$:

→by looking at the diagram, $\mathbf{F_N = 178\ N}$

- Finally, examine *parallel* vectors to find $\mathbf{F_f}$, $\mathbf{F_{Net}}$ and finally **a**:

→since acceleration and $\mathbf{F_{Net}}$ are down-slope,

$\mathbf{F_{Net} = 82.8 - F_f}$ where $\mathbf{F_f = \mu F_N = 0.10(178) = 17.8\ N}$

→so $\mathbf{F_{Net} = 82.8 - 17.8 = 65.0\ N}$

→ $\mathbf{F_{Net} = ma}$ →$\mathbf{65.0 = 20.0a}$ →$\mathbf{a = 3.25\ m/s^2}$

Remember to draw free-body diagrams as you work through <u>any</u> multiple force-related problem; they help in keeping your thoughts organized, particularly as you work through the following examples.

Example #8. A 35 kg mass is set on a smooth inclined surface of μ=0.20. What is its acceleration if $\theta = 34^o$?

(see Dynamics Ex 8 for answer)

Example #9. A 75 kg skier starts down a 30° slope from rest. If the coefficient of friction is 0.10, what is the acceleration <u>and</u> speed 6.0 seconds after starting?

(see Dynamics Ex 9 for answer)

A final note: in these examples where an object is on an incline, the free-body equation always comes down to

$$\mathbf{F_{Net} = mg\sin\theta - \mu F_N}$$

→where $\mathbf{F_N = mg\cos\theta}$ and $\mathbf{F_{Net} = ma}$

→so $\mathbf{ma = mg\sin\theta - \mu mg\cos\theta}$ → masses cancel!

→this leaves $\mathbf{a = g\sin\theta - \mu g\cos\theta}$ → a shortcut for finding **a**.

There are two last aspects to consider in terms of friction: *static* and *kinetic* (sliding).

- **Static friction** is that force used to keep a stationary object from moving; because there is more contact between object and surface, μ_s is large.
- **Kinetic friction** is that force which opposes the motion of an object; since the sliding of object over surface causes less contact between the two, μ_k is small.

It requires more force, therefore, to overcome static friction and *begin* to move a stationary object than it does to keep that object moving against kinetic friction.

Example #10. A muscular physics student needs to move a 145 kg crate across the room over a floor where $\mu_s = 0.370$ and $\mu_k = 0.210$.

a) What minimum horizontal force is required to *just* start the crate sliding?
b) If this force continues to be applied, what will be the rate of acceleration?

(see Dynamics Ex 10 for answer)